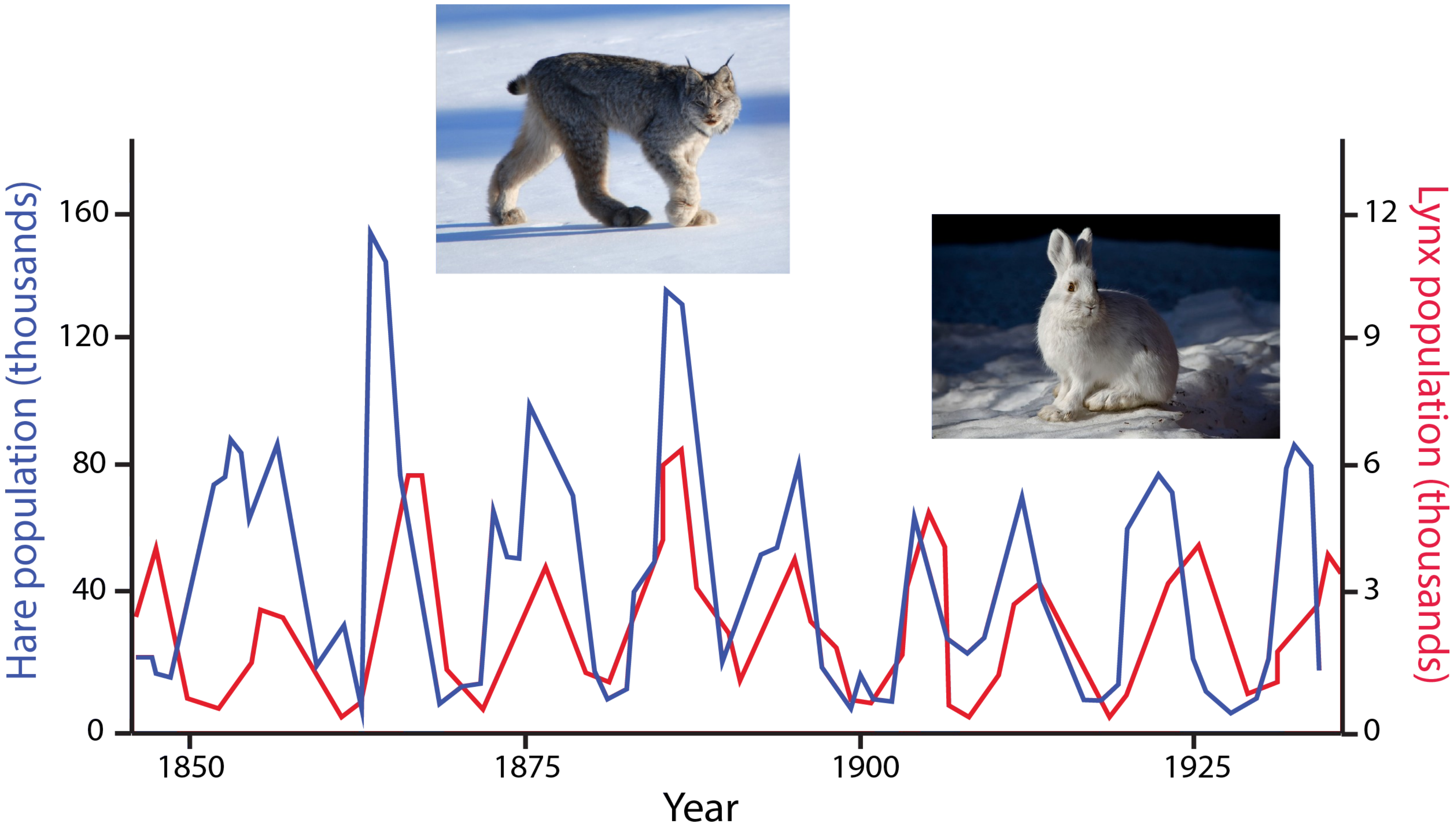
Competition

Predation

Commensalism

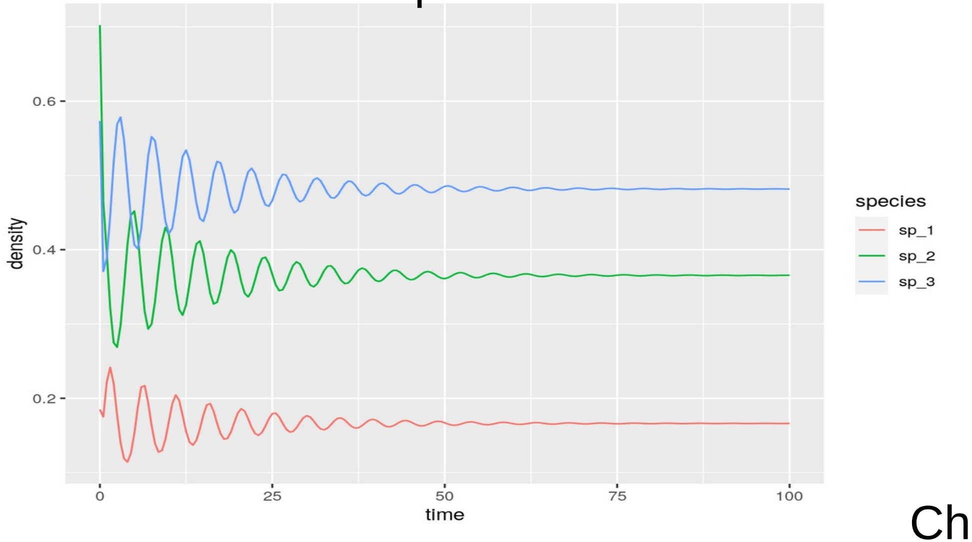
Mutualism



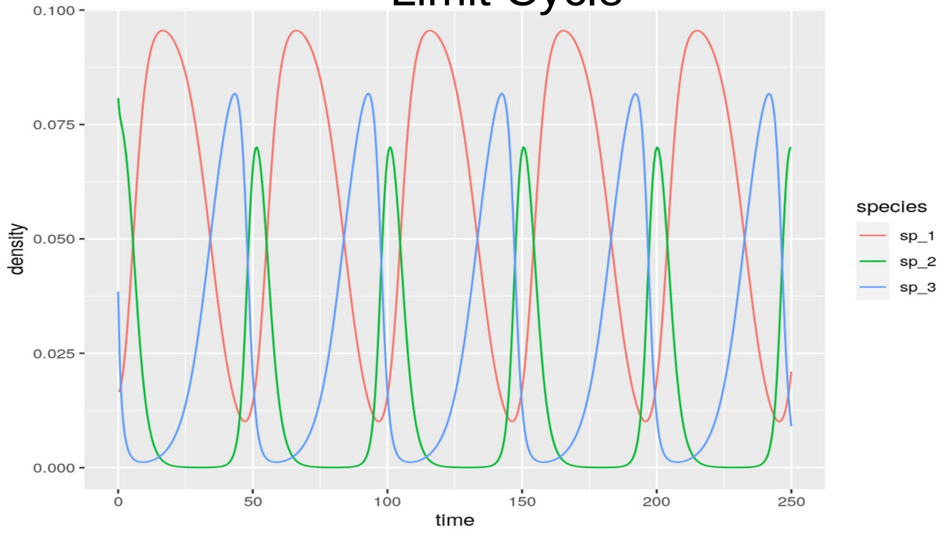
Hare population (thousands)
Lynx population (thousands)
160
120
80
40
0
12
9
6
3
0
1850
1875
1900
1925
Year

Dynamics of species

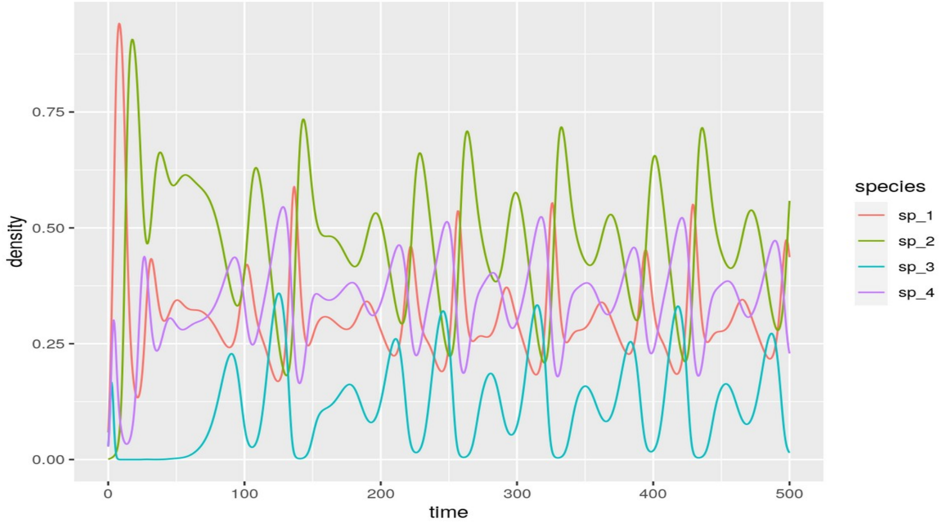
Equilibrium



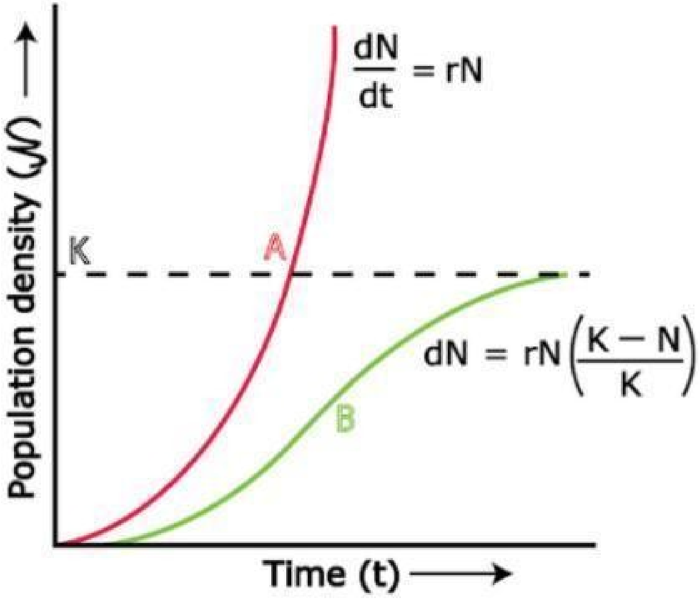
Limit Cycle

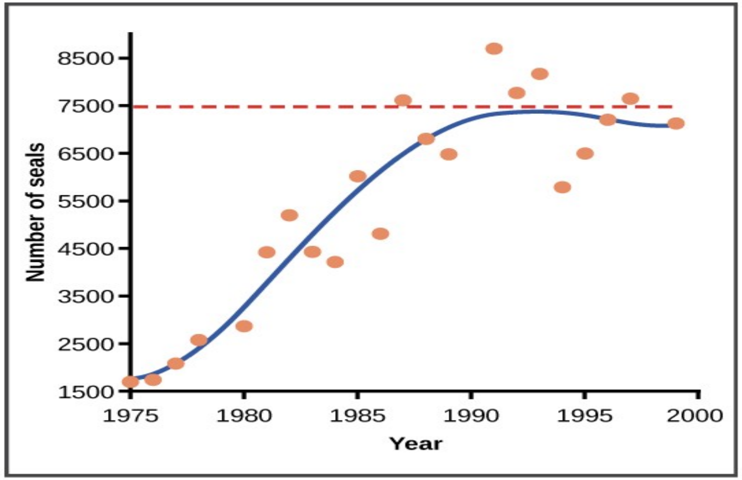


Chaos



Modeling population growth rates





General models for single species populations and analysis of local equilibrium stability

Growth equation for single population : $\frac{dN}{dt}=f(N)$

Algebraic and geometric analysis of local equilibrium stability

Equilibrium point N_0 satisfy $f(N_0)=0$

$$x(t)=N(t)-N_0$$

$$\frac{dx}{dt}=f'(N_0)*x+f''(N_0)*x^2/2+f'''(N_0)*x^3/6+....$$

We obtain an approximate linear differential equation for the perturbation x(t):

$$\frac{dx}{dt}\approx f'(N_0)x$$

$$x(t)=x(0)e^{f'(N_0)t}$$

stable $f'(N_0)<0$

unstable $f'(N_0)>0$

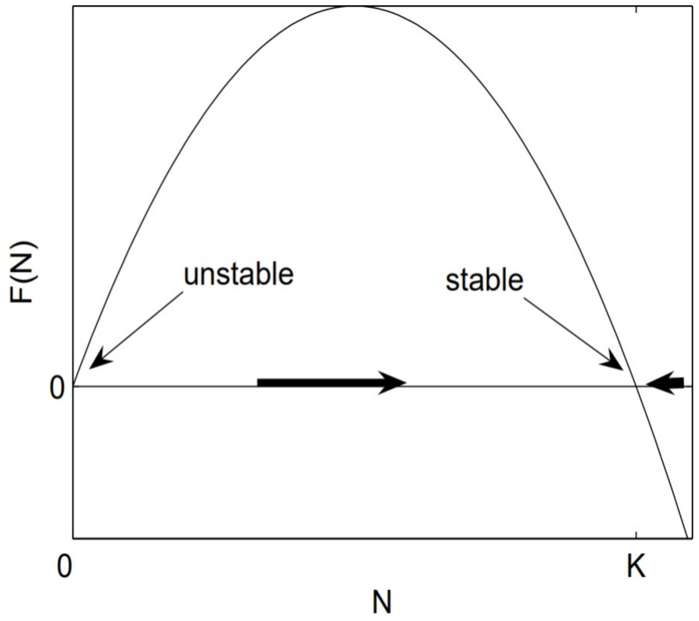
In the case of the logistic equation:

$$f'(N)=r\left(1-\frac{2N}{K}\right)$$ (logistic equation)

Therefore,

$N_0=0$ Unstable equilibrium

$N_0=K$ Stable equilibrium



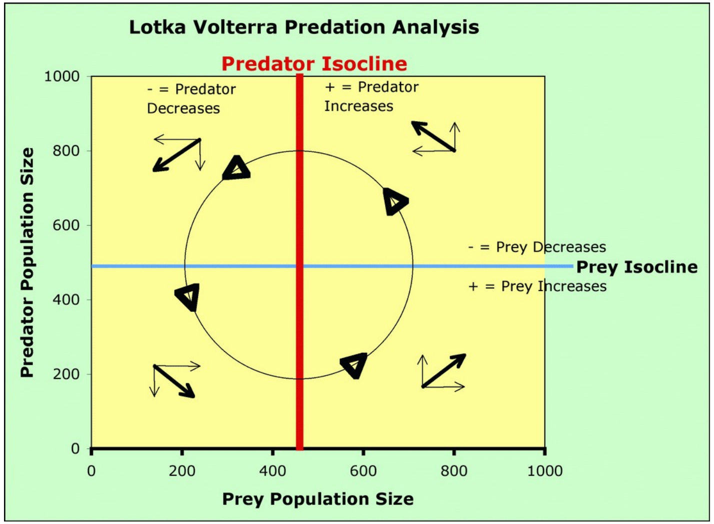
Two-species Lotka-Volterra systems

Predator-Prey Systems

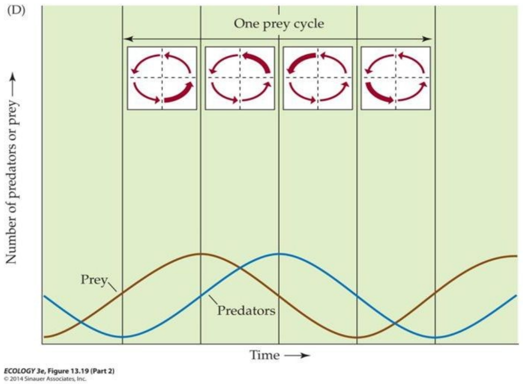
Prey : $$\frac{dN}{dt} = rN - aNP$$

Predator : $$\frac{dP}{dt} = abNP - mP$$

N = number of prey individuals
r = prey per capita rate increase
a = capture efficiency
P = number of predator individuals
m = mortality rate
b = capture efficiency



Oscillations: a mathematical model



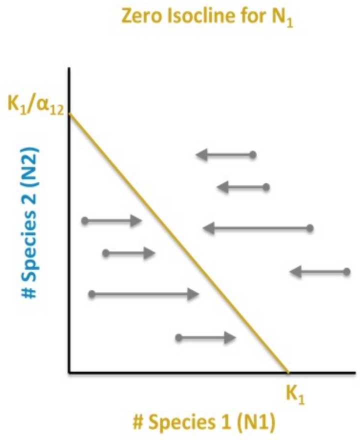
Interspecific competition : Lotka-Volterra Model

a_{12}: The effect an individual of species 2 has on the population growth of species 1

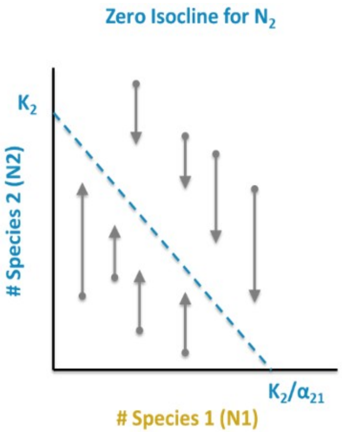
a_{21}: The effect an individual of species 1 has on the population growth of species 2

Species 1 : $$\frac{dN_1}{dt}=r_1 N_1\left(\frac{K_1-N_1-a_{12}N_2}{K_1}\right)$$

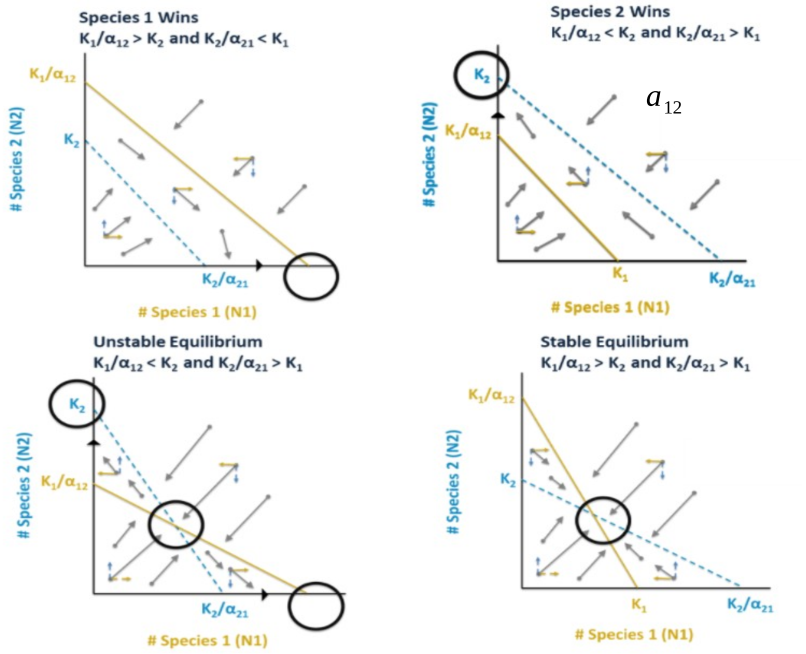
Species2 : $$\frac{dN_2}{dt}=r_2 N_2\left(\frac{K_2-N_2-a_{21}N_1}{K_2}\right)$$



$$N_1 = K_1 - a_{12}N_2$$



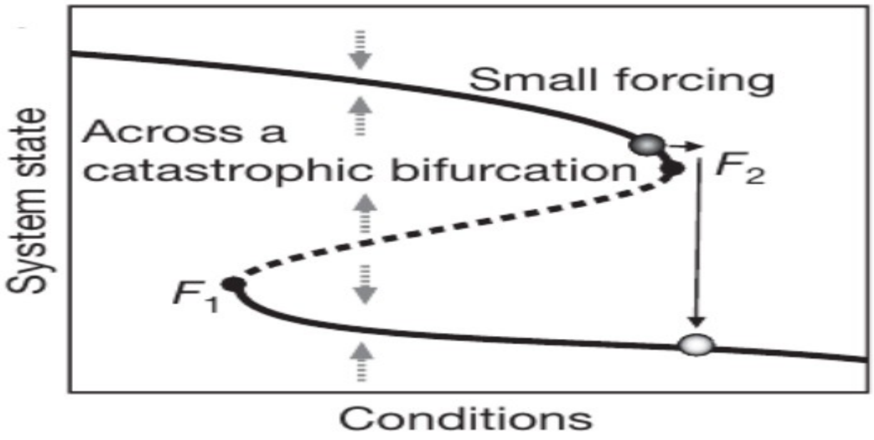
$$N_2 = K_2 - a_{21}N_1$$



What is critical transition or tipping ?

In complex systems, “**tipping**” or “**critical transitions**” are **sudden, large, often irreversible** and usually **unexpected changes** in the state of a system, precipitated under the influence of small stochastic perturbations.

Stochastic Dynamical System



- *Scheffer, M., Bascompte, J., Brock, W.A., Brovkin, V., Carpenter, S.R., Dakos, V., Held, H., Van Nes, E.H., Rietkerk, M. and Sugihara, G., 2009. Early-warning signals for critical transitions. Nature, 461(7260), pp.53-59..*

Mutualism

$$\frac{dN_1}{dt} = r_1 N_1 \left(\frac{K_1 - N_1 + a_{12} N_2}{K_1} \right)$$

$$\frac{dN_2}{dt} = r_2 N_2 \left(\frac{K_2 - N_2 + a_{21} N_1}{K_2} \right)$$

Generalized Lotka-Volterra Model

$$\frac{dN_i}{dt} = N_i \left(r_i + \sum_{j=1}^{n} a_{ij} N_j \right)$$

Mutualistic (+/+) interaction implies both $a_{ij} > 0$ and $a_{ji} > 0$

Competitive (-/-) interaction implies both $a_{ij} < 0$ and $a_{ji} < 0$

Stability of an equilibrium

$$\frac{dN_i}{dt} = f_i(N(t))$$

$$\left.\frac{dN_i}{dt}\right|_{N_0} = f_i(N_0) = 0 \qquad \forall\, i$$

$$\Delta N(0) = N(0) - N_0$$

$$f(\Delta N(0)) = f(N_0) + J|_{N_0} \Delta N(0) + \ldots$$

$$J_{ij} = \frac{\partial f_i(N)}{\partial N_j}$$

We obtain the so-called “community matrix” $M = J|_{N_0}$

$\lambda_{max} < 0$ → $\boldsymbol{N}_0$ is stable

$\lambda_{max} > 0$ → $\boldsymbol{N}_0$ is unstable

For the GLV model, the Jacobian is easy to compute:

$$J_{ij} = \frac{\partial f_i(N)}{\partial N_j} = a_{ij} N_i$$

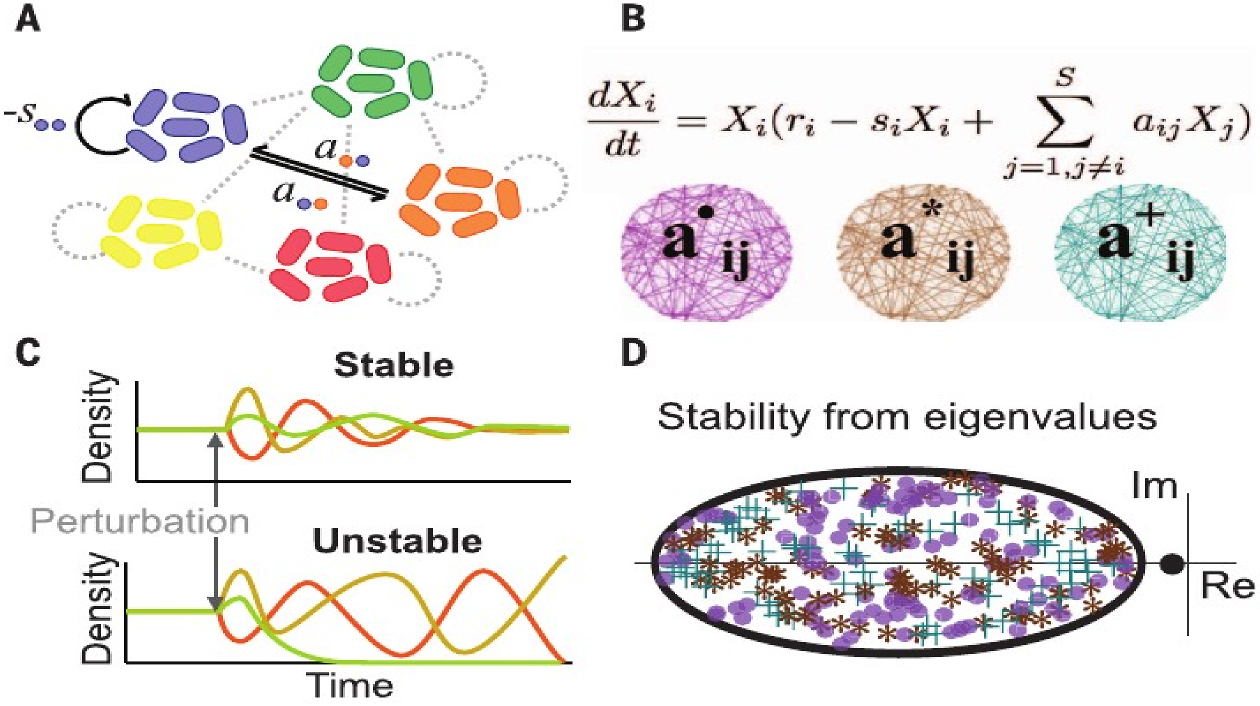
$$J_{ii} = \frac{\partial f_i}{\partial N_j} = r_i + \sum_j a_{ij} N_j + a_{ii} N_i$$

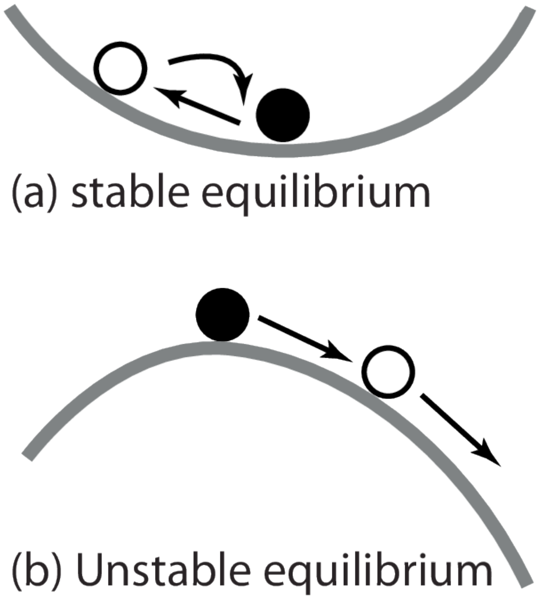
Where nonzero equilibrium point is

$$r_i + \sum_j a_{ij} N_j = 0$$

What is Stability?

Faster the recovery implies higher system stability.
Stability can be analyzed linearly, that is, by studying the eigenvalues of the





Coyte et al., The ecology of the microbiome: networks, competition, and stability, Science 350.6261 (2015): 663-666.

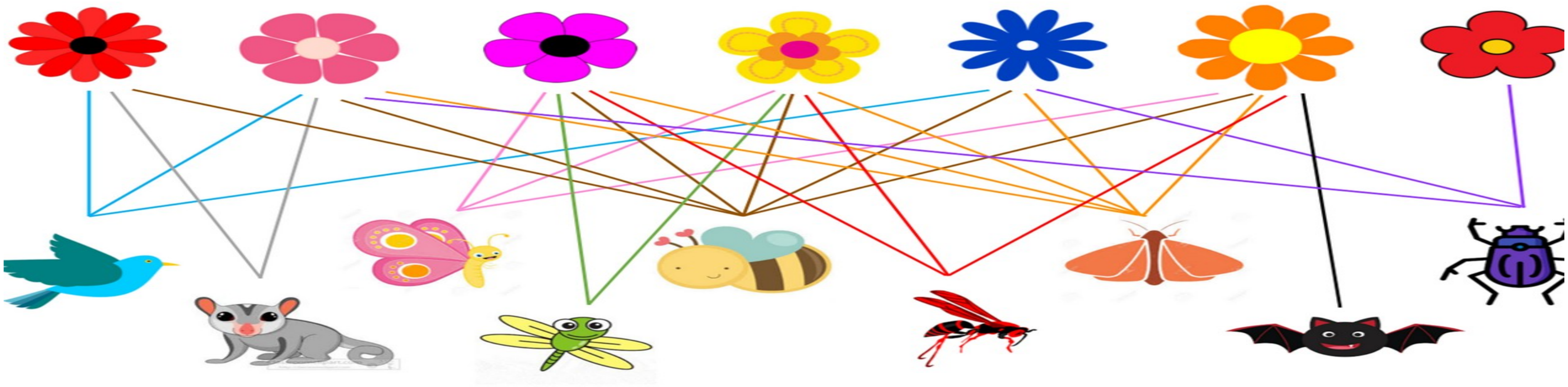
Mutualistic model

Direct competition

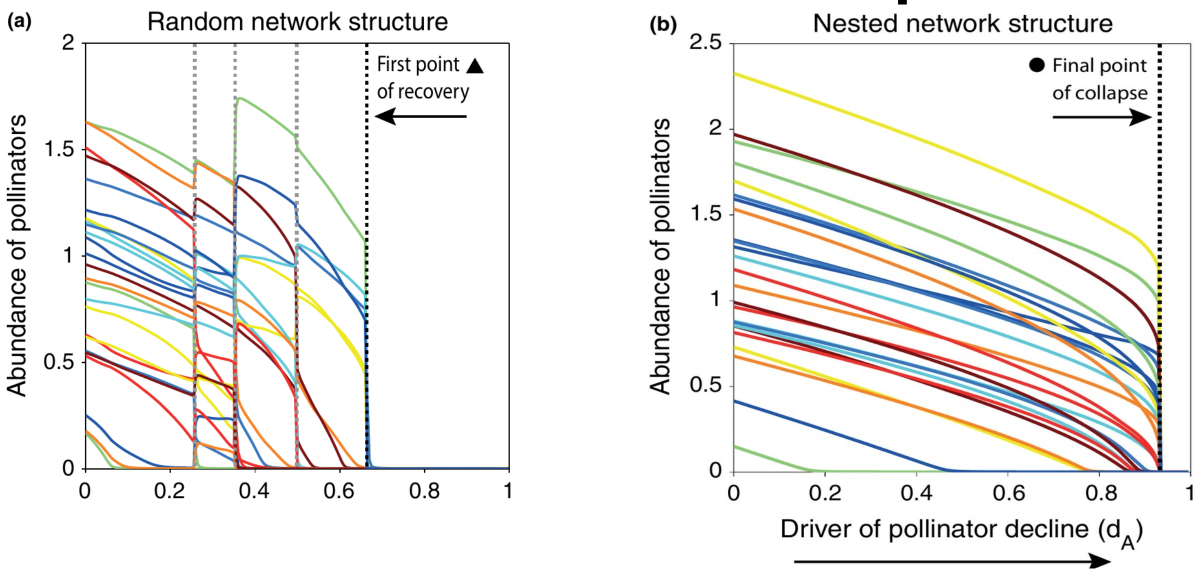
$$\frac{dN_i^{(P)}}{dt} = \alpha_i^{(P)} N_i^{(P)} - \sum_{j \in \mathbf{P}} \beta_{ij}^{(P)} N_i^{(P)} N_j^{(P)} + \sum_{k \in \mathbf{A}} \frac{\gamma_{ik}^{(P)} N_i^{(P)} N_k^{(A)}}{1 + h^{(P)} \sum_{l \in \mathbf{A}} \gamma_{il}^{(P)} N_l^{(A)}}$$

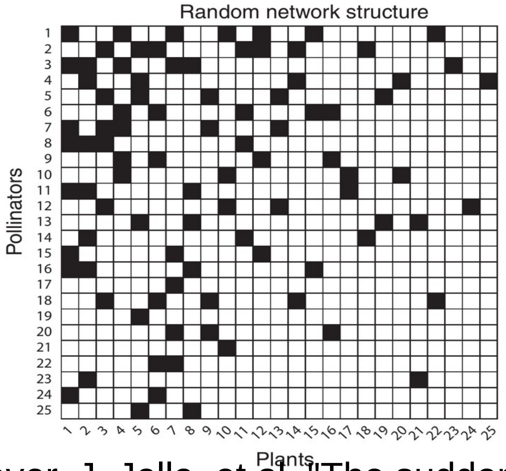
Intrinsic growth rate

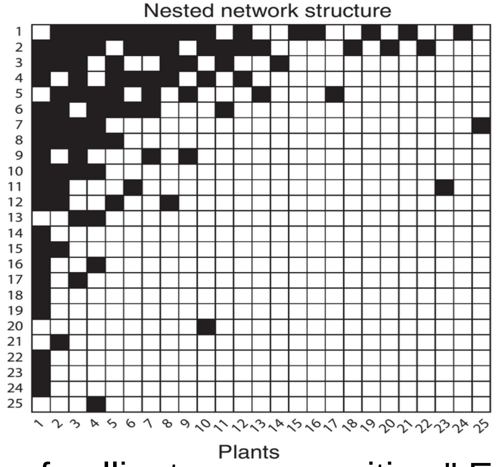
Mutualism

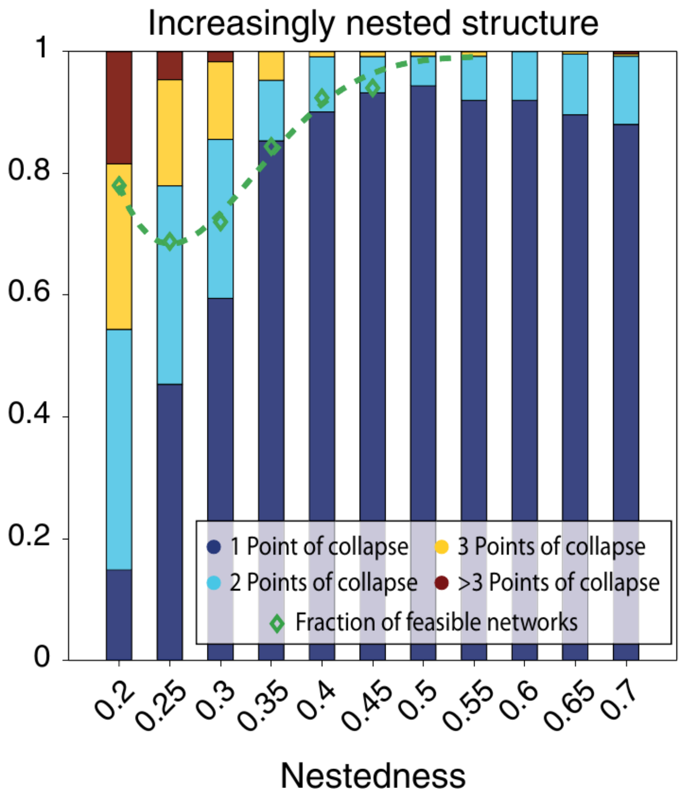


Sudden Collapse in Pollinator Networks



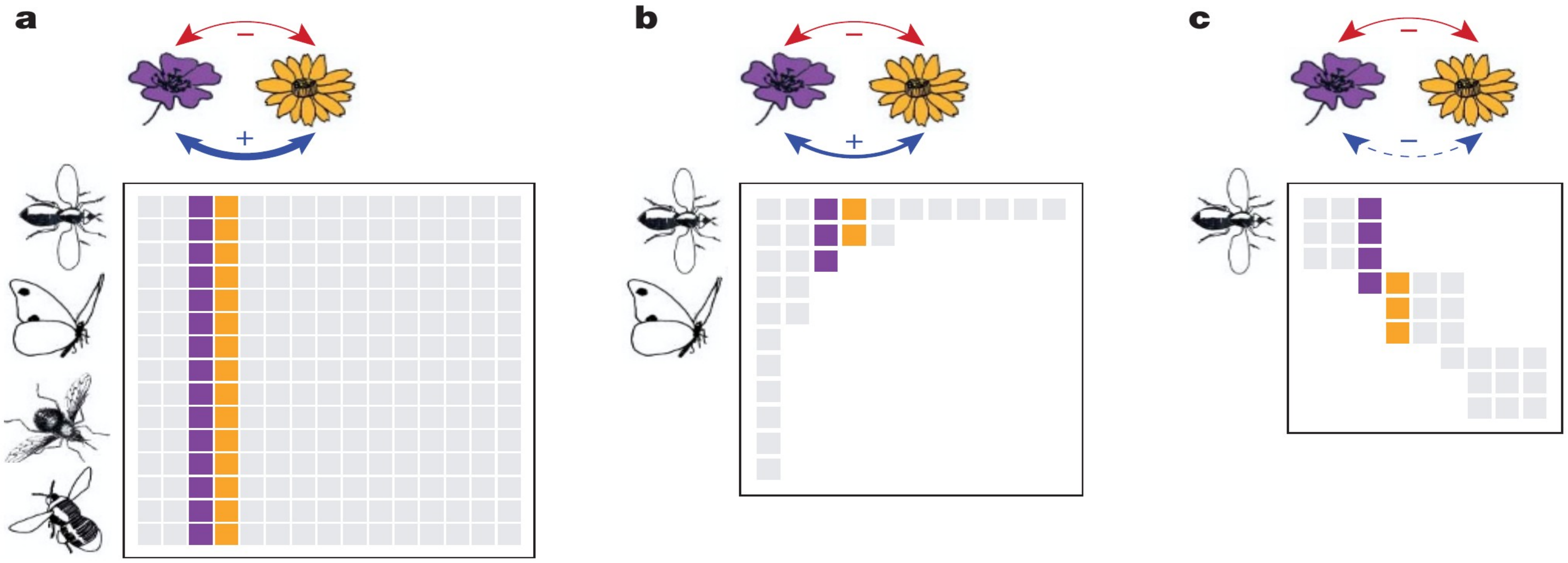






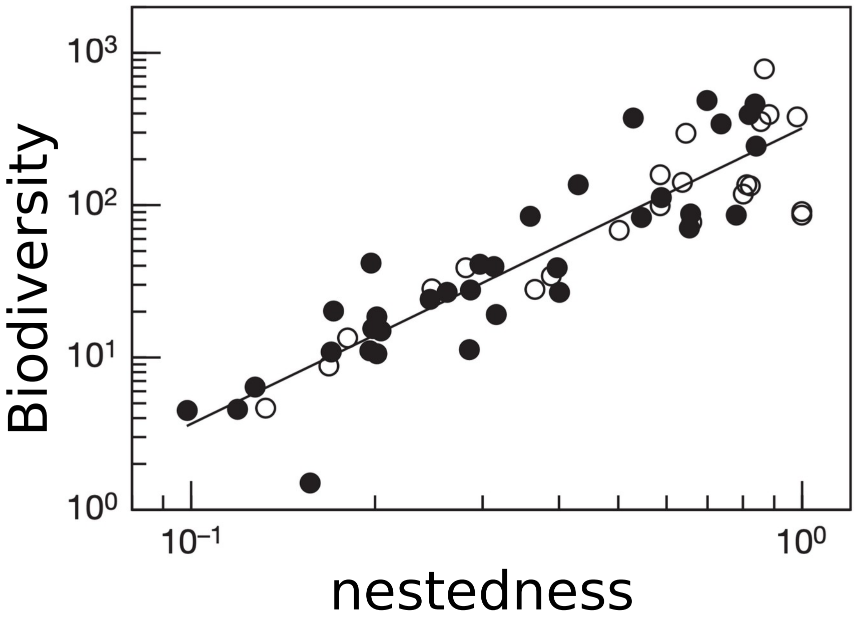
Lever, J. Jelle, et al. "The sudden collapse of pollinator communities." Ecology letters 17.3 (2014): 350-359.

$$C_{ij}^{(P)} = \delta_{ij} + \frac{1}{\bar{S}^{(P)}} + R\left(\frac{1}{S^{(A)} + \bar{S}^{(A)}} n_i^{(P)} n_j^{(P)} - n_{ij}^{(P)}\right)$$

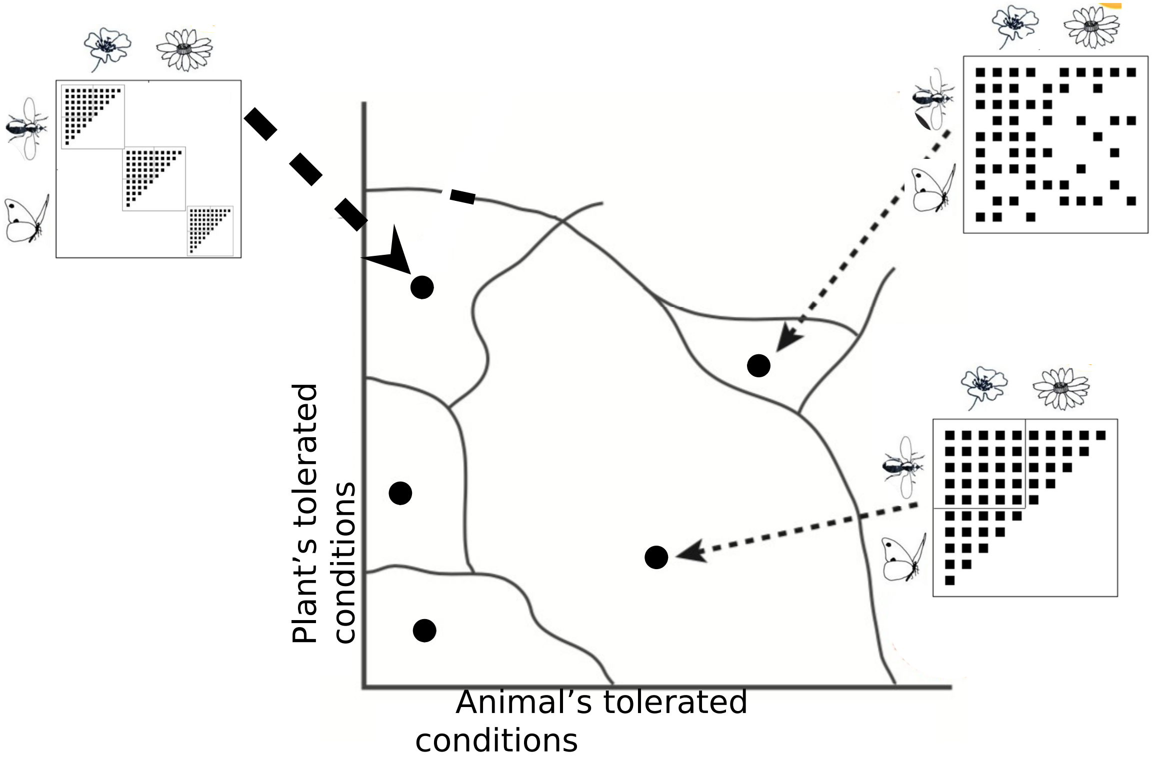


Bastolla, Fortuna, Pascual-García, Ferrera, Luque and Bascompte (2009). *Nature* 458: 1018-1020

Network structure increases robustness

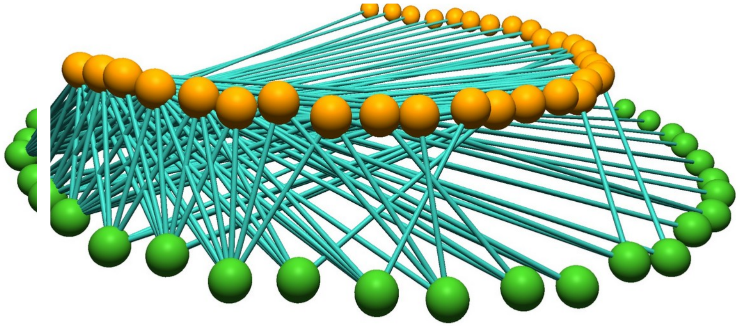


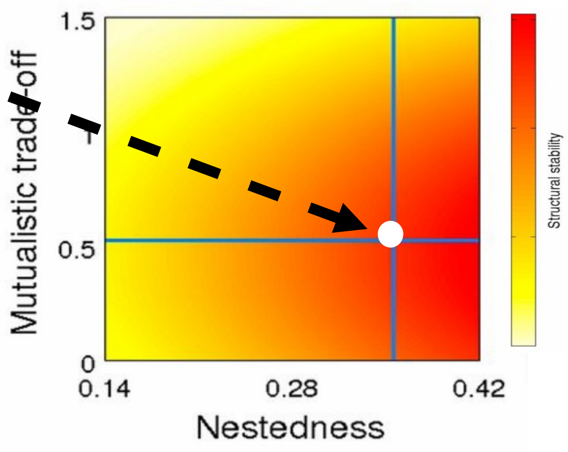
Bastolla, Fortuna, Pascual-García, Ferrera, Luque and Bascompte (2009). Nature 458: 1018-1020



Rohr, Saavedra, and Bascompte (2014). Science, 345: 1253497

Network structure increases robustness





Rohr, Saavedra, and Bascompte (2014). Science, 345: 1253497